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THEORY OF AURORAL MORPHOLOGY

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ABSTRACT

A new theory of auroral morphology based on the production of instabilities in electron sheets is developed. It is found that the theory predicts a longer duration of homogeneous auroral arcs in the early evening, and a shorter duration during morning hours. Observed transitions from glow to homogeneous arcs to ray arcs and draperies are predicted by the theory. An especially interesting feature is the predicted lowering of mirror points accompanying development of brighter features of a display. The theory also achieves encouraging success in explaining horizontal and vertical luminous wave-progressions found in flaming aurora.

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1. Introduction

Theories of the aurora (Störmer, 1955; Chapman and Ferraro, 1940; Alfven, 1950; Chamberlain, 1957, 1960; and others) ascribe auroral forms to energetic charged particles moving in the geomagnetic field. Existing theories do not profess to explain the sequence of events or the various auroral forms appearing in visually observed aurora. Consequently, a theory of auroral morphology has been completely wanting. In the present paper a preliminary outline of such a theory is set forth.

A typical sequence in the appearance of auroral forms has often been remarked (Stagg, 1934, and others). The usual physical sequence is from glow to homogeneous arc, to ray arc, to rays with some intensification before a final breakup of the display occurs. Later the same night the sequence may be repeated. There is therefore a more or less typical morphology of the aurora.

The present theory attempts to explain this sequence on the basis of the stability of groupings of energetic charged auroral particles moving in the geomagnetic field. Also considered are related phenomena such as the polar electrojets of geomagnetism.

2. General Feature of Auroral Morphology

According to Stagg, in latitudes near or just south of the auroral zone a typical display often begins with a glow in the northern sky persisting from minutes to an hour or so (Stagg, 1934). The equatorward region of glow then gradually develops a brightening edge which intensifies into a homogeneous auroral arc. This homogeneous arc next extends roughly nearly from horizon to horizon, with its greatest height roughly above the magnetic meridian. This arc persists without much fluctuation in

intensity perhaps for 15 minutes, and more rarely even for several hours. A transition then occurs, and some irregularities or flutes along the arc may develop, and within a minute or so a transition to a ray arc occurs. The base of the ray arc is usually at a higher level than the homogeneous auroral arc, so that it appears in the low F-region, whereas the homogeneous arc usually has its base in the E-region. Exceptions do occur and rays sometimes penetrate levels as low as 60 km or so. The ray arc is less stable, and it soon breaks up, perhaps with a transient increase in extent and brightness as curtains or draperies. The latter then become dissipated until only shreds of the display may remain as isolated pulsating patches (Stagg, 1934; Vestine, 1944; Störmer, 1955).

More complex situations often occur, many forms appearing simultaneously and at times covering the entire sky. There may also be isolated thin auroral arcs at height 200 km, which extend weakly across the sky for a period of minutes to an hour or so (Störmer, 1955; Vestine, 1943). Isolated red auroral arcs at heights 400 km and above are also known in middle latitudes (Elvey, 1957; Roach, 1960).

3. The Polar Electrojets of Geomagnetism

During magnetic storms, which may occur 10 to 20 times or so a year, and during many lesser periods of disturbances known as magnetic bays, there appear in auroral regions electrojets located within the low ionosphere closely associated with auroral displays. A strong night-time electrojet occurs between local midnight and dawn, with an intense jet of current flowing westward along the auroral zone. A weaker eastward-directed electrojet can often be noted in the afternoon. The current system for a particular bay grows in intensity with time to a maximum and then diminishes

over a period of a few hours. There may be more than one such intensification during a twenty-four hour period. An interesting feature is the tendency for bays to recur at 24-hour intervals for several nights running, a new theory of which will be discussed in subsequent papers.

In recent articles it has been suggested that trapped radiation, moving in accord with predictions based on adiabatic invariants of the particle motion, might form the two electrojets (Vestine, 1960; Chamberlain, Kern, and Vestine, 1960). Although it was suggested that the charged particles responsible were of solar origin, these could without loss of generality be regarded as generated within the geomagnetic field by interactions with solar streams carrying magnetic fields.

A recent suggestion is that geomagnetically trapped particles, when accelerated, drift in longitude about the polar cap along spirals and thus might give rise to a more or less meridional electric field driving the night-time polar electrojet (Chamberlain, Kern, and Vestine, 1960). This explanation has the feature that the Hall conductivity, with a magnitude about ten times that of the conductivity across the geomagnetic field, would dominate in the low ionosphere. The recent studies of auroral arcs and polar electrojets by Akasofu (private communication) have indicated that a meridional electric field may drive the night-time polar electrojet. The aurora and the electrojet are more or less closely associated phenomena (Heppner, 1954). Accordingly, it is of interest to consider interrelated mechanisms (Chamberlain, Kern, and Vestine, 1960). These may predict features expected in the aurora and in the polar electrojets. In particular, it is perhaps constructive to begin by imagining that in the high ionosphere a downcoming filament of trapped radiation aligned

vertically along the geomagnetic field arrives at the night-time electrojet to give rise to an auroral arc. This will next be considered.

If the particle flux producing an auroral display consists of trapped electrons and protons, those not immediately absorbed will drift, electrons to the east and protons to the west, with velocities near the mirror points given approximately by

$$v_d = \frac{3E}{Ber}$$

where E is the energy of the particle, B the magnetic field, e the electronic charge, and r the distance from the earth's center (Christofilos, 1959).

The initial filament a km or so wide, aligned along the geomagnetic field, will produce drift of protons as a thin arc to the west, and electrons to the east.

One possibility is that charges may accumulate near mirror points within the atmosphere, producing electric fields with associated current flows in the ionosphere. In the case where particles are accelerated, the charge separation on the night side may give a meridional electric field in the ionosphere. In this case, a situation favorable to generation of the night-time electrojet may arise.

The details of the electric field distribution that could give rise to the polar electrojet system during magnetic bays are unknown. The eastward-directed electrojet flowing in the low ionosphere on the early evening side, can be driven by an electric field from west to east in the low ionosphere, where the electrical conductivity has been enhanced during the day, with conductivity augmented by the penetration of auroral particles during an auroral display. At a slightly higher level, the electric field during the electrojet would be more effective if applied in

a poleward direction. If it be supposed that the west-to-east electric field at lower levels is the more important, this electric field E or its motor effect will tend to cause the trapped radiation producing the jet to drift poleward also.

In the case of the early-morning electrojet, an electric field directed from pole to equator locally is regarded as the more likely driving force, since the Hall electric conductivity within the E-region is greater than that for current flow parallel to the electric field and across the geomagnetic field.

4. Stability of Homogeneous Auroral Arcs

Students of mirror machines and plasma physics have learned a few facts about the stability of trapped radiation (Kruskal and Schwarzschild, 1954; Post, 1956; Rosenbluth, 1960; Rosenbluth and Longmire, 1957). It is of course not necessarily known that auroral morphology arises as an analogous consequence of plasma instabilities arising from radiation trapped in the geomagnetic field. An auroral arc, once formed, however, provides a region of increased particle flux which should spread rapidly eastward (or westward) to extend the arc, as noted previously. If homogeneous auroral arcs appear simultaneously in both northern and southern hemispheres, joined by geomagnetic field lines, the base of the arc in either hemisphere may be described as anchored within electrically conducting levels of the ionosphere. If, as supposed here, there is connection of an auroral arc in the northern hemisphere via geomagnetic field lines with an arc in the southern hemisphere, physical experience with mirror machines leads one to expect that it is only a question of time until fluted irregularities in auroral formations will appear

(Kruskal and Schwartzchild, 1954 and others), but the greater the E-region conductivity, the greater this time should be. These flutes grow rapidly and the display breaks up. The importance of the application of these theoretical concepts to the great natural phenomenon of the aurora seems to have been suggested only recently (Vestine and Sibley, 1960).

It has already been remarked that in the case of polar electrojets in the low ionosphere associated with auroral displays, the Hall electric conductivity in the region of current flow is probably about ten times the cross conductivity. Consequently, the electrojets are likely to be driven by nearly meridional electric fields, regarded as arising between drifting proton and electron sheets oriented more or less vertically and east-west. Attention here will be confined to the sheet closest to the equator. This will here be identified, tentatively, as having a homogeneous auroral arc at its base, in which, any minute irregularities should grow exponentially with time, in accordance with the principles and methods developed by Kruskal and Schwarzschild in their classic paper, and by Rosenbluth and Longmire (1957).

First the stability of an idealized homogeneous auroral arc will be investigated. The main assumption is that a northern auroral zone arc is sustained by geomagnetically trapped particles. The region of particle incidence is taken as a thin sheet oriented roughly east-west in a surface of constant integral invariant (parallel to geomagnetic field lines) and connected via the magnetic field with a similar arc in the southern hemisphere. An initial supply of trapped electrons penetrating beneath the ionosphere will drift east, and any protons west, with separation of the charges. This will occur both in the northern and southern hemispheres.

The width in latitude l and length L of field lines joining the northern and southern hemispheres may be of order of say 400 km and 10^5 km, respectively. The ionospheric circuit, as in mirror machine studies, will comprise resistance R overall capacitance C , and a current I .

If flutes involving charge irregularities are formed on a longitudinal sheet of incident particles, such as the diamagnetic sheet of section 4, perturbation electric fields will appear in the plane of the incident particle sheet. Such perturbation electric fields will produce drift normal to the plane of the sheet. Thus if the incident particle sheet is represented by the xz -plane, drift may occur in the y -direction such that $\dot{y} = E'/B$ where \dot{y} is the drift velocity, E' is the perturbation electric field, and B the local magnetic field. It is assumed that collisions can be neglected, as in the case in the F-region.

Writing δ for the distance between equivalent perturbations in the x -direction (east-west), the perturbation electric field E' can be replaced by equivalent potential perturbations $V' = B\delta\dot{y}$. Now the east-west drift of particles inside the sheet provides a westward directed current $I = 2 L \rho \bar{v}_d y$, where y is the north-south sheet thickness, L is the length of a field line, connecting arcs in the northern and southern hemispheres, ρ is the space charge density (given by $n_e e = -n_i e$), and \bar{v}_d is an average transverse drift velocity for the particles moving in Störmer orbits along the magnetic field lines. This current links the regions of charge density variation.

A circuit equation for the perturbation voltage V' can therefore be written

$$\dot{V}' + \frac{V'}{CR} - \frac{I}{C} = 0 \quad (1)$$

or noting the resistance $R = \delta / \sigma_1 A$, where σ_1 is the transverse electric conductivity, and A is the cross-sectional area of the part of the incident particle sheet of finite thickness within the ionosphere; the cross-section is that imagined obtained by a vertical slice taken roughly in the north-south direction. Then

$$\sigma_1 A B R \ddot{y} + (\sigma_1 AB/C) \dot{y} - (2L \rho \bar{v}_d/C) y = 0 \quad (2)$$

so that

$$y = \alpha \left[\exp(4L \rho \bar{v}_d / \sigma_1 AB) t - \exp(-2\sigma_1 A/C)t \right] \quad (3)$$

where α is a constant. It can be shown that the second term, under the circumstances, rapidly decays, so that y grows exponentially with the time at a rate which is less when the resistance $1/\sigma_1 A$ is less.

An early evening homogeneous auroral arc often lasts about 15 minutes (~ 1000 seconds) before breaking up. During the morning hours, however, the direct conductivity appearing in equation (3) may be reduced by a factor of 10 or more. Thus (3) predicts increased rates of perturbation buildup during morning hours. Störmer (1955) gives observational data that indicates that morning displays tend to be more transitory in character than those appearing earlier in the evening.

A characteristic time constant for a perturbation buildup from equation (3) can be calculated. The time for an e-fold increase of y is given by

$$\tau = \sigma_1 AB / 4 L \rho \bar{v}_d \quad (4)$$

Taking $\sigma_1 = 10^{-14}$, $A = 2 \text{ km} \times 50 \text{ km}$, $B = 0.4$, $L = 10^5 \text{ km}$, $\bar{v}_d = 10^{-3} \text{ km/sec}$ (for, say, 6 kev electrons), and $\rho = ne = 10^2 \times 1.6 \times 10^{-20} \text{ emu/cm}^2$, we obtain from equation (4) $\tau = 600 \text{ sec}$. This seems compatible with observed

durations of homogeneous auroral arcs. It should be noted that this requires a continuous supply of 6 kev electrons, since nearly all particles of this energy will be absorbed at auroral levels, without reflection (Welch and Whitaker, 1959).

The homogeneous auroral arc is usually replaced by a ray arc, with the lower end of the illumination or rays in the low F-region. At this level σ_1 is considerably less than in the E-region, say by a factor of ten, so that the time constant τ is less, or perhaps 60 seconds. In the case of draperies the height of the base may be even higher, so that breakup of the displays may occur with an even shorter time constant. It may also be remarked that auroral rays penetrating to levels below 80 km, where the electric conductivity is also relatively small, seem to have a short duration. The short-circuiting properties of the electrically conducting atmosphere therefore seems to exert an important stabilizing influence, by reducing the local electric field irregularities which tend to grow and disrupt the displays. Consequently, the transition sequence in auroral forms and in particular the time of ending of auroral forms seems dependent on the stability of displays, which in turn tends to be greater when disrupting electric fields can be short-circuited by a good electrical conductor.

5. Generation of an Auroral Arc by Geomagnetically Trapped Radiation

A broadly distributed auroral glow, perhaps associated with incoming protons, often precedes a brighter auroral display within the glow itself. The glow is often followed in a limited region by an intensification of illumination that may be caused by a local influx of charged particles. This possibly means there is drainage of trapped particles due to a local

lowering of the mirror point of incident radiation in the brighter region. This brightening region may develop on the equatorward edge of the glow. The cause of such mirror point depression is not known. One suggested mechanism is that solar streams locally stretch or compress geomagnetic field lines so that particles may drift equatorwards near midnight, to lower mirror point heights when certain invariants of the particle motion are conserved (Vestine, 1960). Once established, such an auroral form, limited in extent, will probably have a lifetime closely related to the local electric conductivity which short circuits any electric field irregularities in the display (Vestine and Sibley, 1960). As mentioned above, this may explain why auroral displays in the early morning appear to be shorter in duration than in early evening; in fact, in the evening, the auroral glow preceding the brighter form may suggest a buildup in the electric conductivity permitting a longer display lifetime. This lifetime may be affected by the events in the local irregularity or ridge in an east-west direction referred to above. Thus a discharge to lower levels, through secondary effects of electric and magnetic fields set up in the region, may cause growth or accentuation of the discharge to lower reaches of the ionosphere. This may arise as a consequence of local acceleration of the trapped particles, promoting increased flux penetration to auroral levels. The factors leading to the transition from glow to the first indications of a homogeneous auroral arc are of course obscure, and as yet quite unknown. Since such transitions often occur, certain electrical and magnetic effects associated with the production of the first bright segment of the homogeneous auroral arc will be considered here. In fact, this observed feature of a brightening edge of illumination

of the glow on the low mirror-point equatorward-side would be expected either as a result of a slight acceleration of the particles, or equatorward migration of a glow produced by trapped radiation.

Diamagnetic Sheet Mechanism. If charged particles can be supplied to a region at auroral heights at such a rate that a net space charge is built up, associated electric and magnetic fields will arise. Drift of particles at right angles to the electric field of the space charge distribution, and to the ambient magnetic field, leads to reduced magnetic field intensities on the interior of the charge distribution. Such a polarization distribution may occur at the base of a homogeneous auroral arc.

It is therefore of considerable interest to estimate the magnetic field B within the interior of a simple charge distribution, maintained by continuous supply of incident particles, and to try to assess the implications for auroral morphology.

An increase of trapped particle flux density due to an injection event over a given longitude will be regarded as the source of incident particles. In order that there be growth of a space charge, the flux must consist predominantly of particles of one sign (for example, electrons). The overall space charge distribution is to some extent neutralized by small motions of charges in the surrounding partially ionized upper atmosphere.

The space charge distribution considered here is oriented in a surface of constant integral invariant $I = \int_{B_m}^{B^*} \sqrt{1 - B/B_m} \, dl$ in the geomagnetic field B (Vestine and Sibley, 1960). Such a surface will be elongated approximately along lines of geomagnetic latitude. The thickness of the sheet will be taken as small compared to its longitudinal extent. Suitable reference axes are provided by taking the origin of right-handed Cartesian

coordinates at the top of the sheet with the z-axis downward; the y-axis is normal to the sheet, and the x-axis internal to the sheet and horizontal. For purposes of comparison with the orientation of an auroral arc due to incident electrons in the northern hemisphere, z is in the direction of the geomagnetic field, x is to the east and tangent to the mirror point curve for particles of fixed energy drifting along a curve of constant integral invariant (Vestine and Sibley, 1960).

Suppose that the incident flux is sufficient to maintain a space charge distribution ρ such that $\rho(x, z) = \text{constant}$ and $\rho(y)$ is given by $\rho = \rho_0 \cos (\pi/b)y$, where b is the thickness of the charge distribution and the distribution is symmetric about the plane $y = 0$. The particle drift velocity magnitude v is approximately equal to E/B ; this drift will be in the x-direction, the approximation being very good in the F-region, and fairly good down to upper reaches of the E-region, where the collisional frequency becomes comparable with the spiral frequency of electrons.

From Poisson's equation

$$\nabla^2 V = \frac{d^2 V}{dy^2} = -4\pi c^2 \rho_0 \cos (\pi/b)y = -\frac{dE}{dy} \quad (5)$$

where V is the electric potential, c the velocity of light and b the thickness of the sheet. Thus

$$E = 4 c^2 \rho_0 b \sin (\pi/b)y \quad (6)$$

may provide a useful approximation of physical interest. Since the distribution is symmetric about $y = 0$, and the charge drift direction is opposite on either side of this plane, the magnetic field produced by the

moving charges opposes the original ambient field B_0 inside the charge distribution. This decrease in the magnetic field lowers the mirror points for trapped radiation mirroring inside the charge distribution.

The magnetic field decrease dB due to current flow in a sheet of thickness dy at a distance y from the center of the charge distribution, is approximately (except near end points) $dB_g = 2\pi \rho v dy = 2\pi \rho (E/B) dy$, or roughly $2\pi(\rho/B_0) E dy$, if B_0 is the field in the absence of the currents, the field decrease due to these currents is small compared with B_0 . Noting that v reverses on crossing the plane $y = 0$, the field on the central plane of the charge sheet $B(0)$ can be written

$$B(0) = B_0 - \frac{8 c^2 \rho_0^2 b^2}{B_0} \quad (7)$$

Consideration of the contribution of regions $(-y, y)$ and $(-b/2, y)$, $(y, b/2)$ leads readily to the change in field as a function of y

$$\Delta B(y) = -(8 c^2 \rho_0^2 b^2 / B_0) \cos^2 (\pi/b)y \quad (8)$$

Thus B is reduced inside the sheet. For this reason, the particle flux increases, since particles mirroring at higher altitudes (lower field intensities) can now penetrate the region. Hence ρ_0 increases, and b decreases, so this theory may provide some insight respecting the remarkable thinness often remarked in auroral arcs. The effect is reminiscent of the pinch effect in charged particle beams, but here the effect is due to lowering of mirror points (Bennett, 1934). It is not known whether or not transitions in auroral form from a diffuse glow over a broad area to a thin arc can be explained using this mechanism. More detailed study is desirable on this point. In the meantime, however,

the reduction of b and increase in ρ_0 , clearly predicted by the foregoing analysis, are in encouraging agreement with observation.

The space charge density used above can be considered in a dynamic sense as the number of particles per cm^3 times the particle charge. The contribution to n depends upon source, upon particles mirroring in a given region, and upon particles traversing the region (Welch and Whitaker, 1959); also included are particles coming to rest in the region as a result of inelastic collision processes. In the case of an isotropic 6 Kev electron flux of the order of 5×10^{10} particles $\text{cm}^{-2} \text{ sec}^{-1} \text{ steradian}^{-1}$ (McIlwain, 1960), n is about 10^2 per cm^3 . ΔB is very sensitive to the thickness b . However, for $b = 4\text{km}$, $B_0 = 0.450$, and $n = 10^2$, then $\Delta B = -8c^2 e^2 n^2 b^2 / B_0$, so $\Delta B = -0.0074 \text{ gauss} = -740 \text{ }^\circ$. This corresponds to a change in mirror point altitude $(0.0074/0.450)r_0 = 110 \text{ km}$, permitting deeper penetration of incident particle flux. Since $\Delta B \sim 4\pi b \sigma_2 E = 4\pi n e v b \sigma_2 / \sigma_1$, where σ_2 is the Hall conductivity, when the flux $n e v \sim \sigma_1 E$ (the loss), a homogeneous arc may be lowered to maximum σ_2 / σ_1 , at height about 110 km.

The observable effect of the diamagnetic sheet mechanism would be a transition from a quiescent broad glow pattern to a thin auroral arc with ionization drifting at high velocity parallel to the arc surface and normal to the geomagnetic field. For space charge distributions due to electron flux, the drift will be opposite that for protons.

The composition of the incident flux may therefore be expected to contribute to drifts in auroral rays, which may in part explain observational data on drift of ionization at auroral altitudes (Nichols, 1957) as well as frequently observed visible drifts in aurora (Becken and Maehlum, 1960). The broad features of the present diamagnetic sheet may

be identified with the auroral arcs and their morphology. In particular, it will be instructive to consider the stability of such auroral arcs in another way, and consider whether evidence of such instability exists. This matter will next be considered, neglecting other features of the diamagnetic sheet, such as the fact that the mirror points of the down-coming spiralling radiation are supposed within the electrically conducting ionosphere, so that the space charges rapidly dissipate. It will however, be assumed that the rate of supply of charge is adequate to maintain the system.

6. Development and Stability of Draperies and Ray Arcs

If the diamagnetic sheet model described above in 4 and 5 is assumed, electron beam theory may be of special interest in interpreting features of ray and drapery structures which develop from an initial homogeneous auroral arc. Of such beam investigations, the experimental studies by Webster (1955), Kyhl and Webster (1956), and Cutler (1956) show instability patterns in sheet electron beams extremely suggestive of auroral behavior. Alfven (1950) suggested this application of electrodynamics to drapery structures, but theoretical interest has apparently been centered around electron beam tube development. Pierce (1956), and Kyhl and Webster (1956) treat the problem of the hollow cylindrical electron beam, developing stability criteria.

The following treatment aims to apply and extend the theory in connection with the sheet beam described in 4. It is first shown that elementary considerations of a thin sheet beam aligned along the geomagnetic field leads to a plausible explanation of observed sheet beam instability. If there is an electric field due to the space charge of the beam, drift

motion of the electrons in the crossed electric and magnetic fields will lead to growth of any charge density perturbation.

The general effect of an electric field perturbation due to a charge perturbation is illustrated in Figure 1, as a drift motion caused by the field perturbation. It is seen that the drift distorts the sheet beam into an "S" curve. This leads to a further perturbation of the electric field. As a consequence, the drift motion enhances the original charge perturbation, and charge perturbations propagate along the beam at right angles to the magnetic field. The progression of this distortion leads ultimately to vortical regions of high particle flux, connected by thin filaments (Kyhl and Webster, 1956).

Early stages of this process can be identified with auroral rays involving distortions of a sheet beam of incident particles (Alfven, 1950).

Consider the theory of a sheet beam of the kind indicated in 4, for reasons of theoretical convenience being supposed of zero thickness. Any waves associated with finite beam thickness are therefore neglected. A periodic boundary condition is obtained by specifying $n+1$ regions of variation for the sheet length $n\delta$ where δ is the distance separating regions of variation.

Suitable solutions giving the perturbations or displacements in a plane sheet auroral arc which vary with time and distance in the x-direction (direction of east-west drift motion) and z-direction (direction of downward particle propagation along the geomagnetic field B) are conveniently regarded as a product of typical terms

$$e^{i\omega t} e^{-i\beta z} e^{-i\gamma x}$$

where $\gamma = 1/\delta$, with δ real (Pierce, 1956).

$$\text{Let } W = \omega - \beta u_0 - \gamma u_1$$

where u_0 and u_1 are the average particle velocities in the z- and x- directions, respectively.

The equations of motion are

$$\begin{aligned}\ddot{x} &= i W \dot{x} = \frac{e}{m} \frac{\partial V}{\partial x} + \omega_c \dot{y} \\ \ddot{y} &= i W \dot{y} = \frac{e}{m} \frac{\partial V}{\partial y} - \omega_c \dot{x} \\ \ddot{z} &= i W \dot{z} = \frac{e}{m} \frac{\partial V}{\partial z}\end{aligned}\tag{9}$$

where e/m is taken as positive and $\omega_c = Be/m$ is the spiral or gyro frequency.

The equation of continuity must be satisfied for particle flow within the auroral arc. That is, the divergence of the current density is equal to the negative of the time rate of change of the charge density in the arc. Let ρ be the charge density of the auroral sheet of thickness b and ρ_0 the average value of ρ . Then

$$i\omega\rho = i\beta(\rho u_0 + \rho_0 \dot{z}) + i\gamma(\rho u_1 + \rho_0 \dot{x})\tag{10}$$

and

$$\rho = (\beta\rho_0/W) \dot{z} + (\gamma\rho_0/W) \dot{x},\tag{11}$$

with ρ_0 negative. Consequently the fluctuations in charge density depend both on frequency and displacement velocity. Following Pierce (1956), if B_z is the magnitude of the magnetic field at the surface of the beam, and u_1 the drift velocity, the surface electric field E_y is $-\partial V/\partial y = u_1 B_z$.

The potential due to the thin auroral arc of thickness b will be assumed to be of the form

$$V_1 e^{-ky} - (\alpha b p_0 / \epsilon) y, \quad y > 0 \quad (12)$$

$$V_2 e^{ky} + (1-\alpha) b p_0 / \epsilon y, \quad y < 0$$

where α is a constant, and ϵ the dielectric constant.

From Poisson's equation $k^2 = \beta^2 + \gamma^2$, and there must also result the relation

$$V_1 - V_2 = (b p_0 / \epsilon) y \quad (13)$$

where y is a small displacement in the y -direction. From Gauss's theorem, to the first order,

$$-k (V_1 + V_2) = (b p_0 / \epsilon) \quad (14)$$

Hence

$$V_1 + V_2 = \frac{1}{k} \left[\frac{\beta b p_0}{W} \dot{z} + \frac{\gamma b p_0}{W} \dot{x} \right] \quad (15)$$

Thus

$$\begin{aligned} -\frac{\partial V}{\partial y} &= -\frac{k}{2} (V_1 - V_2) = -\frac{k b p_0}{\epsilon} y \\ -\frac{\partial V}{\partial z} &= i(\beta^2 u_1 B / kW) \dot{z} + i(\beta \gamma u_1 B / kW) \dot{x} \\ -\frac{\partial V}{\partial x} &= i(\beta \gamma u_1 B / kW) \dot{z} + i(\gamma^2 u_1 B / kW) \dot{x} \end{aligned} \quad (16)$$

Substituting the above in the equations of motion (9) and eliminating velocities (together with $k^2 = \beta^2 + \gamma^2$) yields

$$(W^2 - k u_1 \omega_c)(W^2 + k u_1 \omega_c) = \omega_c^2 \left[W^2 - (\beta^2 / k) u_1 \omega_c \right] \quad (17)$$

In cases of physical interest in connection with aurora

$$1/4 \gg k^2 u_1^2 / \omega_c^2 - \beta^2 u_1 / k \omega_c \quad (18)$$

hence

$$W = \pm \omega_c \left[1 + \frac{k^2 u_1^2}{\omega_c^2} - \frac{\beta^2 u_1}{k \omega_c} \right]^{1/2} \quad (19a)$$

$$W = \pm i \omega_c \left[\frac{k^2 u_1^2}{\omega_c^2} - \frac{\beta^2 u_1}{k \omega_c} \right]^{1/2} \quad (19b)$$

(19a) gives, very nearly, $W = \pm \omega_c$, which is of no physical interest in the present context.

(19b) represents the condition for growing waves when

$$k^3 u_1 > \beta^2 \omega_c \quad (20)$$

$$\text{Now } W = \omega - \beta u_0 - \gamma u_1 = \omega - \beta u_0 - u_1 / \delta$$

For stationary waves $\omega = 0$, and

$$\beta = -\frac{1}{\delta} \frac{u_1}{u_0} \pm i \omega_c \left[\frac{k^2 u_1^2}{\omega_c^2} - \frac{\beta^2 u_1}{k \omega_c} \right]^{1/2} \quad (21)$$

The wavelength of such waves propagating in the z-direction will be

$$L = -(\text{Re} \beta)^{-1} = \delta u_0 / u_1 \quad (22)$$

The time constant for buildup in the z-direction is given by

$$\tau = -(\text{Im} \beta)^{-1} = \frac{1}{\omega_c} \left[\frac{k^2 u_1^2}{\omega_c^2} - \frac{\beta^2 u_1}{k \omega_c} \right]^{-1/2} \quad (23)$$

where it is assumed that $\text{Re}\beta \gg \text{Im}\beta$, and β^2 is taken as real. We find that τ can also be written

$$\tau = \frac{1}{ku_1} \left(1 - \frac{\beta^2 \omega_c^2}{k^3 u_1^3} \right)^{-1/2} \quad (24)$$

Letting $-\beta = (u_1/\delta u_0)$ as an approximation, and with $\gamma \gg \beta$, we have $L \gg \delta$ and $k \approx \gamma = 1/\delta$. This implies

$$\tau = \frac{\delta}{u_1} \left[1 - \frac{\delta u_1 \omega_c}{u_0^2} \right]^{-1/2} \quad (25)$$

For the earth's field at auroral heights, take $B = 0.45$, so that $\omega_c = 8 \times 10^6 \text{ sec}^{-1}$, $\delta = 3 \times 10^5 \text{ cm}$ (from observations), and suppose $u_1 = 6 \times 10^4 \text{ cm/sec}$ (for ionization drift velocities) within a typical fluted auroral structure. Thus instabilities will develop in a time $\tau = 50(1 - 1.44 \times 10^{17}/u_0^2)^{-1/2}$, if $u_0^2 > 15 \times 10^{16}$, or $u_0 > 4 \times 10^8 \text{ cm/sec}$. This corresponds to electron energies of about 40 eV.

The diamagnetic sheet mechanism of 4 shows that u_1 is determined by the space charge distribution, and observed values of u_1 suggest values of E smaller than the model value. For a given structure (δ and b given) u_0 will have an upper limit for stability which depends upon electron mobilities, and thus is related to the conductivity at auroral altitudes.

Since in a diamagnetic sheet the electric driving force yielding the drift velocity u_1 increases with increasing charge density ρ , decreased conductivity reduces the loss of ρ with time, or tends to enhance u_1 with time. From (25) since the growth time τ is shortened when u_1 is increased, decreased electric conductivity leads to increased instability

of the display. This accords with various experimental observations of electron beams (Cutler, 1956).

It is also interesting to note that from (25), u_0^2 must exceed $\delta \omega_c u_1$ in order that τ be real, thereby promoting growth of instabilities. Consequently the stability is sensitive to increase in the incoming particle velocity u_0 .

From the foregoing, we see that elementary considerations of electron beam theory predict perturbations of a thin auroral arc. The first stage in perturbation growth resembles very closely auroral drapery structures. Perturbation growth proceeds to ray-like patterning in which the charge distribution as indicated by experiment (Kyhl and Webster, 1956; Cutler, 1956) is vortical about magnetic field lines. Because of higher mirror points of the remaining particles the base of a ray arc is higher than in a homogeneous arc

7. Comparison with Observation

Auroral Features Before and After Midnight. It has frequently been remarked that homogeneous auroral arcs of longer duration are a feature of the hours before magnetic midnight and that displays thereafter are more transitory and fleeting in space and duration. Since the ionosphere decreases in electric conductivity during the night, the theory of stability of homogeneous auroral arcs would appear to predict this outstanding feature of auroral morphology.

Radar Observations of Auroral Drift. Investigations of auroral reflections using HF and VHF radar (Nichols, 1957; Unwin, 1959; Lyon and Kavadas, 1958; Bullough and Kaiser, 1957) indicate motion of reflecting regions, westward prior to magnetic midnight and eastward following magnetic midnight. These data were obtained using antennas with broad radiation patterns. Leadsbrand, Presnell, Berg and Dyce (1959), using UHF narrow

beam-width radar, have obtained Doppler-shift patterns indicating predominantly westerly drift motions, independent of time of day, with average velocities of 500 m/sec. This UHF data gave similar results for both diffuse and discrete auroral forms. The diamagnetic sheet mechanism developed above predicts drift directions within auroral forms due to incident electrons observed to the north corresponding to those inferred by Leadabrand et al (1959), but opposite results for the poleward side of the diamagnetic sheet.

Leadabrand, et al (1959) further found that east-west echo drift of the reflecting region aggregate was not systematic (although the 500 m/sec Doppler shift drift was predominantly westward), with eastward or westward drifts equally likely. Diffuse auroral echoes rarely exhibited any motion and most of the time appeared to come from the underside of a layer extending as much as 800 km in magnetic latitude and 500 km east-west at about 90-110 km altitude (lower in the north). Discrete echoes were usually reflected from the same region as the diffuse. The discrete echoes corresponded to auroral arcs along lines of geomagnetic latitude. The motion of the sheet as a whole, as inferred from the diamagnetic sheet mechanism, would be controlled by the broad pattern of local electric and magnetic fields arising from ionospheric current systems, as Nichols (1957) has suggested.

It may well be that many features of auroral drift motions are explained by the diamagnetic sheet model, since it moves readily in response to variations in incident particle flux, to the local electric fields, and to magnetic gradients associated with ionospheric current systems. The latter may play a part in the formation of pairs of homogeneous auroral arcs joined at one end (Vestine, 1960). The high east to west UHF doppler shift drifts of Leadabrand, et al (1959) can be interpreted on the basis

of the drift velocities internal to the sheet. Observational data such as those of Kim and Currie (1958) do not afford indications of the drift velocities internal to the sheet.

Observations of Auroral Form Transitions. Stoffregen et al (1960), reporting on studies of D-region ionization during auroras in northern Sweden, find that such ionization is closely associated with sudden increases in auroral visual intensity. They infer that both primary electrons of energy in excess of 100 kev and secondary x-rays contribute to D-region ionization. They ascribe such high energy radiation to leakage from the Van Allen belts (Arnoldy, Hoffman and Winckler, 1960). The auroral forms preceding the sudden intensity increases were typically G, HA, HB, and R moving slowly toward lower latitudes. Auroral intensity increased slowly and smoothly for about 1/2 to 2 hours, then suddenly increased in intensity to a high value. Following the sudden increase the aurora were much brighter, varying rapidly in visual intensity, and covered great parts of the sky, moving south. Red color appeared, followed by pulsating forms ("flaming aurora"). It is noted that the observations seem to accord well with the present theory, the increased brightness and pulsations being predicted as a manifestation of instability before breakup. The diamagnetic sheet mechanism offers a possible means for tapping high mirror point trapped radiation, accentuating the brightness and hastening the breakup.

Observations of Pulsating or Flaming Aurora. Oscillatory phenomena are commonly observed in auroral displays. The most spectacular are described as so-called "flaming aurora" which usually develop immediately preceding the end of displays. Other modes of pulsation have been reported, however. Geddes (1939) reports three pulsating arcs in which waves were observed

traveling along the arcs following one another at intervals of about ten seconds. The wave phenomena immediately preceded the RA phase and stopped before development of RA. Using the results of the electron beam instability theory, we would deduce drift velocities of $u_1 = \delta/\tau$ where δ is the separation of regions of charge density perturbation and $\tau = 10$ sec as given by Geddes observations. Suppose $\delta = 5$ km is the separation of elements of the developed RA, then $u_1 = 5 \times 10^4$ cm/sec. This is in agreement with drift velocities obtained from radar doppler shift measurements.

These results for flaming aurora (F), with vertical propagation of waves, as described in the Photographic Atlas of Auroral Forms (1930), are predicted by the present theory for auroral morphology. According to Elvey (private communication), he and his coworkers have also considered the flaming aurora as a manifestation of plasma instabilities.

In the unusual flaming aurora observed by Geddes (1939) in New Zealand, the wave movement involved the progress of a bright HA from a southern elevation of 15° to 50° in about $1/2$ second, followed by subsequent sequences of similar arcs. If a distance of 300 km is taken from observer to HA, the distance traversed is about 200 km. This gives a wavelength L of about 400 km (peak intensity to peak intensity) with a period τ of $1/2$ second for buildup of the disturbance. It would therefore be inferred that incident particle velocity $u_0 = L/\tau$ is about 10^8 cm/sec. This corresponds to electron energies somewhat less than the 6 Kev reported by McIlwain (1960) from direct measurements in a bright HA.

Vestine (1944) reports a vertically propagating wave display. In a flaming aurora associated with HB, the wavelength L determined from

his data is about 25 km with period τ of about 0.002 second (observed over a span of about 12 waves). This gives the incident particle velocity u_0 as about 1.2×10^9 cm/sec, in fair correspondence to McIlwain's (1960) direct HA electron energy measurements. Near the end of the display, the wavelength had about doubled, the period was about half the original, and the HB had disappeared. Using the electron beam instability theory, it would be predicted that the incident particle velocity u_0 is about 4 times the original or about 10^{10} cm/sec. This corresponds to electron energies of about 100 Kev as inferred by Stoffregen, et al (1960) in their D-region ionization studies.

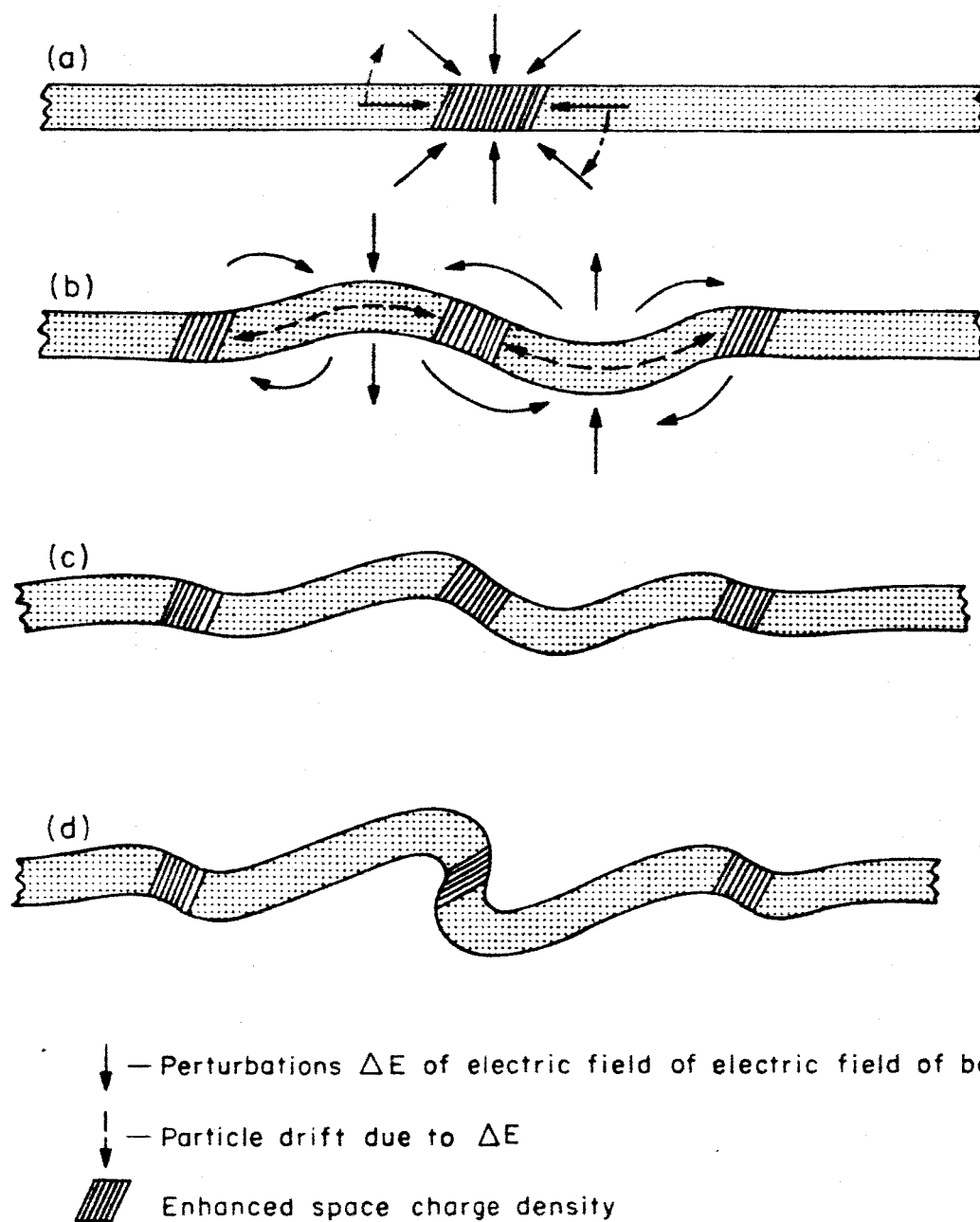


Fig. 1—Mechanism of disturbance buildup in thin auroral arc

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